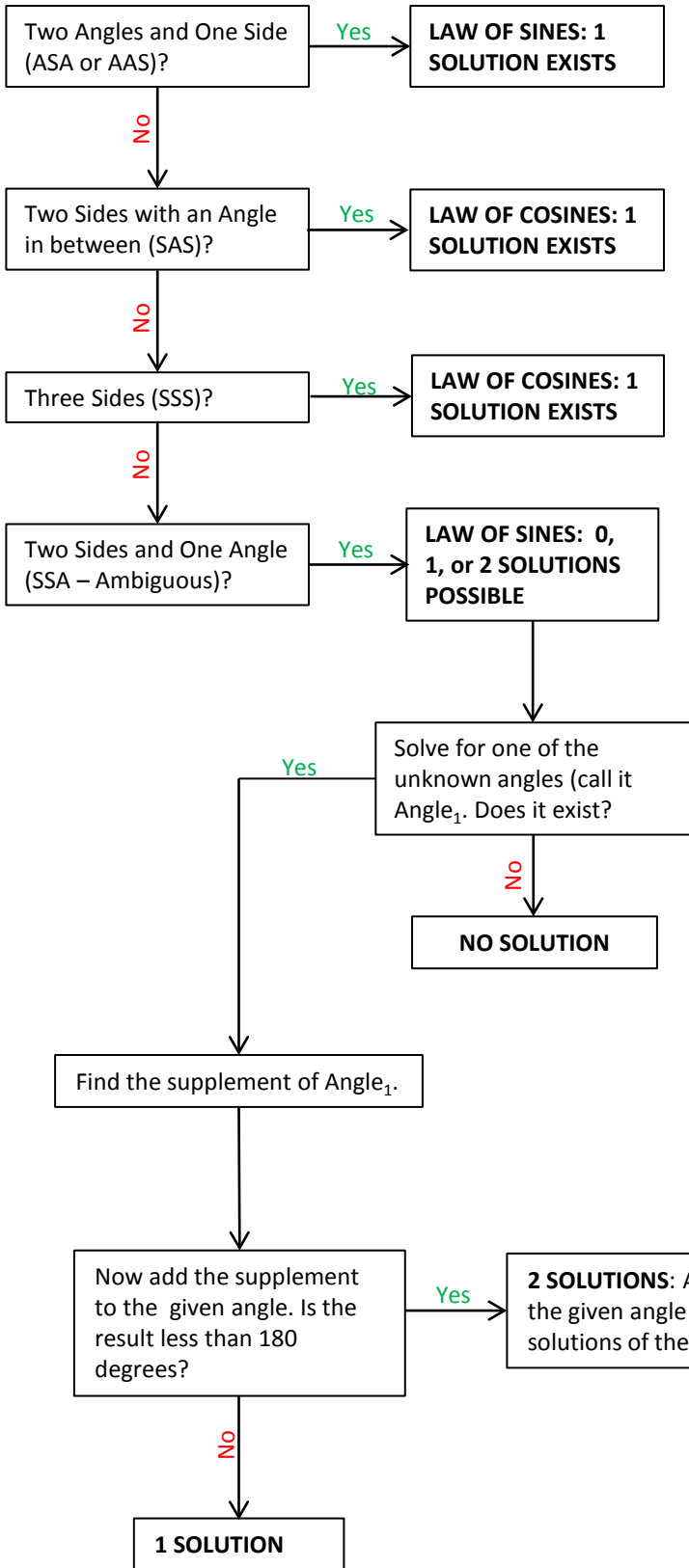


LAW OF SINES AND COSINES

Do you have...



LAW OF SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

LAW OF COSINES

These equations are used for solving sides

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

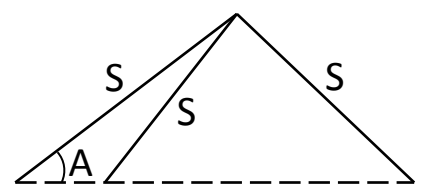
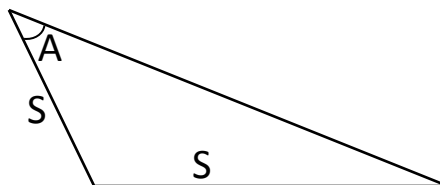
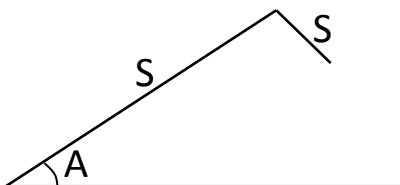
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Variations (used for solving angles):

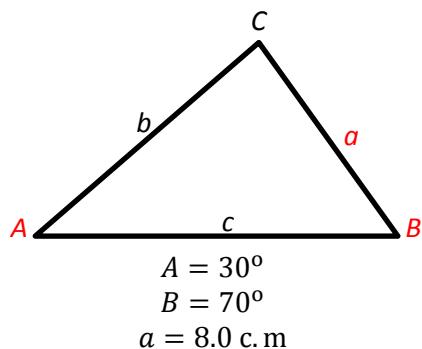
$$A = \cos^{-1} \left(\frac{b^2 + c^2 - a^2}{2bc} \right)$$

$$B = \cos^{-1} \left(\frac{a^2 + c^2 - b^2}{2ac} \right)$$

$$C = \cos^{-1} \left(\frac{a^2 + b^2 - c^2}{2ab} \right)$$



Law of Sines Example:



$$A + B + C = 180^\circ$$

$$C = 180^\circ - 70^\circ - 30^\circ$$

$$C = 80^\circ$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 30^\circ}{8.0 \text{ c.m.}} = \frac{\sin 80^\circ}{c}$$

$$c = \frac{(8.0 \text{ c.m.})(\sin 80^\circ)}{\sin 30^\circ}$$

$$c \approx 16 \text{ c.m.}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 30^\circ}{8.0 \text{ c.m.}} = \frac{\sin 70^\circ}{c}$$

$$c = \frac{(8.0 \text{ c.m.})(\sin 70^\circ)}{\sin 30^\circ}$$

$$c \approx 15 \text{ c.m.}$$

Solution:

$A = 30^\circ$	$a = 8.0 \text{ c.m.}$
$B = 70^\circ$	$b \approx 15 \text{ c.m.}$
$C = 80^\circ$	$c \approx 16 \text{ c.m.}$

Law of Sines Example: (Ambiguous Case)

$$C = 35.4^\circ$$

$$a = 205 \text{ units}$$

$$b = 314 \text{ units}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{205} = \frac{\sin 35.4^\circ}{314}$$

$$\sin A = 0.3782$$

$$A \approx 22.2^\circ$$

$$A' = 180^\circ - 22.2^\circ$$

$$A' = 157.8^\circ$$

$$C + A' = ?$$

$$C + A' = 35.4^\circ + 157.8^\circ$$

$$C + A' = 193.2^\circ$$

Only one solution exists

$$A + B + C = 180^\circ$$

$$22.2^\circ + B + 35.4^\circ = 180^\circ$$

$$B \approx 122.4^\circ$$

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

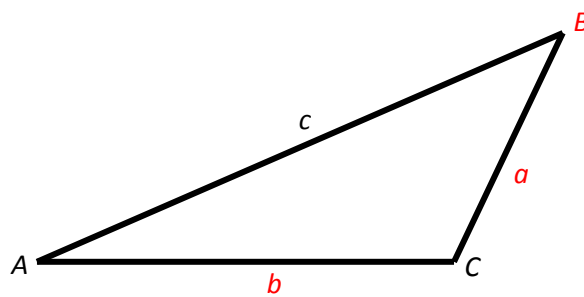
$$\frac{\sin 122.4^\circ}{6} = \frac{\sin 35.4^\circ}{c}$$

$$c = \frac{314 \sin 35.4^\circ}{\sin 122.4^\circ}$$

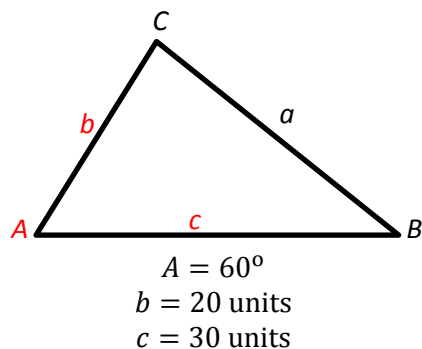
$$c \approx 458$$

Solution:

$A \approx 22.2^\circ$	$a = 205 \text{ units}$
$B \approx 122.4^\circ$	$b = 314 \text{ units}$
$C = 35.4^\circ$	$c \approx 458 \text{ units}$



Law of Cosines Example:



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 20^2 + 30^2 - 2(20)(30) \cos 60^\circ$$

$$a^2 = 400 + 900 - 1200(0.5)$$

$$a = \sqrt{700}$$

$$a \approx 26.46$$

$$B = \cos^{-1} \left(\frac{a^2 + c^2 - b^2}{2ac} \right)$$

$$B = \cos^{-1} \left(\frac{700.1316 + 900 - 400}{2(26)(30)} \right)$$

$$B \approx 39.7^\circ$$

$$A + B + C = 180^\circ$$

$$60^\circ + 39.7^\circ + C = 180^\circ$$

$$C \approx 80.3^\circ$$

Solution:

$A = 60^\circ$	$a \approx 26 \text{ units}$
$B \approx 42^\circ$	$b = 20 \text{ units}$
$C \approx 78^\circ$	$c = 30 \text{ units}$